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APPROXIMATE SOLUTIONS FOR FLIGHT-PATH ANGLE OF A REENTRY VEHICLE IN THE UPPER ATMOSPHERE

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SUMMARY

Two basic equations for approximating the flight-path angle of a reentry vehicle in the upper atmosphere are derived from the equations of motion. These solutions for flight-path angle are obtained (1) by assuming constant velocity over the portion of the trajectory under consideration, and (2) approximating the velocity value for use in the development of the equation for flight-path angle.

Solutions obtained from numerical integration of the nonlinear reentry equations are used to evaluate how well the derived equations approximate the true value of flight-path angle. The trajectories selected were such that the space vehicle stayed within a safe flight corridor. Reentry trajectories for a high-drag low-lift vehicle included in the investigation covered a range of velocities from suborbital to hyperbolic, altitudes from 200,000 feet to 300,000 feet, and desired flight-path angles from 0° to 2°. In the case of a high-lift reentry vehicle, trajectories that obtained level-flight conditions at velocities ranging from suborbital to near-orbital velocities for an altitude of 200,000 feet were selected to evaluate the closed form equations.

INTRODUCTION

For an atmospheric reentry at supercircular velocity and landing at a prescribed point on the earth's surface, precise guidance of a space vehicle is required. This guidance must be effected in the upper atmosphere to bleed off excessive velocity before either (1) descending directly to the landing point, or (2) performing a skipout maneuver to extend the range.

Numerous practical reentry guidance systems have been formulated in recent years by experimental and analytical procedures. Many of these workable guidance systems, such as those based on rapid time of predictions and those utilizing reference trajectories and their adjoint solutions require a complex computer. A guidance system based on a closed form solution of the equations of motion would be more attractive because of the reduction in computer storage and computer component requirements. Because of the nonlinear nature of the equations of motion, closed form solutions for complete entries cannot be

obtained; however, approximate relations for the trajectory variables can be obtained for selected portions of an entry trajectory.

Two procedures where closed form solutions are utilized for specific maneuvers are discussed in references 1 and 2. In reference 1, a space vehicle is controlled to prescribed exit conditions from the atmosphere to achieve a desired range. In reference 2, the vehicle is controlled to level-flight conditions at a desired altitude. In other studies in which approximate solutions to reentry equations are derived (for example, refs. 3, 4, and 5) certain assumptions are made such that the closed form equations obtained are valid for approximating quantities such as deceleration, total heat absorbed, heating rate, and so forth, over a fairly large interval of time. The present study was made in order to develop a closed form solution for flight-path angle as a function of altitude and desired end conditions that could satisfactorily approximate flight-path angle over a short period of time - 1 minute or less.

In the development of the guidance system reported in reference 2, an equation for approximating flight-path angle was derived and used to predict when a control maneuver should be made to bring a reentry vehicle to level flight at a specific altitude. The present investigation evolved from this phase of reference 2. The original equation for approximating flight-path angle was modified and other equations were derived.

In the present paper, trajectories calculated from the equations of motion are used to determine the relative accuracy of the closed form equations. Trajectories covering a wide range of conditions in the upper atmosphere, that is, velocity, altitude, and flight-path angle, were selected in order to determine where these approximations are valid. It should be noted that the range of parameters used in this study is chosen within a previously established flight corridor.

SYMBOLS

A constant used to simplify the second term of equation (16)

B constant used in exponential approximation of atmospheric density, ft

c constant used in the development of equation (17)

C_L lift coefficient

C_D drag coefficient

g acceleration due to gravity

h altitude above earth's surface, ft

$\Delta h = h_d - h$, ft

K_1, K_2 constants defined in equations (2) and (3)

K_3 constant used in the development of equation (10)

L/D   lift-drag ratio

m   mass of vehicle, slugs

n   integer

r   radius of earth, ft

S   surface area of vehicle, sq ft

t   time, sec

V   velocity of vehicle, ft/sec

V_1 constant mean value of velocity, ft/sec

w_1, w_2, x, z variables used in development of equation (17)

W   weight of vehicle, lb

$y = e^{-h/B}$

γ flight-path angle, deg or radians

γ_1 first approximation of flight-path angle, deg or radians

$$\epsilon = \int \frac{gB}{V^2}\left(1 - \frac{V^2}{gr}\right)\frac{dy}{y}$$

ρ_c constant used in exponential approximation of atmospheric density, slugs$/$ft^3

Subscript:

d   values at desired conditions

     Derivatives with respect to time are denoted with dots over the variables.

ANALYTICAL DEVELOPMENT

     For the analysis made in the present study, it was assumed that the earth is spherical with a radius of 20,908,800 feet. A constant gravitational field

of 31.2 ft/sec^2 was assumed for all the altitudes covered. It was also assumed that the earth is stationary in all respects and there is no relative motion of the atmosphere.

From the equations of motion, two basic closed form solutions for flight-path angle are obtained. In the first, constant velocity is assumed. In the second, an approximate solution for velocity is obtained and used to develop the closed form equation for flight-path angle. These two methods are further modified as is subsequently shown.

Equations of Motion

With these assumptions, the two-degree-of-freedom equations of motion of an entry vehicle as derived in reference 6 are as follows:

$$\dot{h} = V \sin \gamma \tag{1}$$

$$\dot{V} = -K_1 V^2 e^{-h/B} - g \sin \gamma \tag{2}$$

$$V\dot{\gamma} = K_2 V^2 e^{-h/B} - g \cos \gamma \left(1 - \frac{V^2}{gr}\right) \tag{3}$$

where

$$K_1 = \frac{\rho_c S C_D}{2m}$$

$$K_2 = \frac{\rho_c S C_L}{2m}$$

A standard substitution used to make the equations more concise is:

$$y = e^{-h/B}$$

Also, in the present investigation, the small-angle approximation of

$$\sin \gamma = \gamma$$

$$\cos \gamma = 1$$

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is used. Hence, equations (1) to (3) become:

$$\dot{y} = -\frac{y}{B} V\gamma \tag{4}$$

$$\dot{V} = -K_1 V^2 y - g\gamma \tag{5}$$

$$V\dot{\gamma} = K_2 V^2 y - g\left(1 - \frac{V^2}{gr}\right) \tag{6}$$

In general, in other studies velocity has been considered the independent variable. (See, for example, ref. 3.) However, for path control velocity variation is of secondary importance to the variation of flight-path angle and altitude. Accordingly, in this study, altitude is made the independent variable.

Dividing equations (5) and (6) by equation (4) gives

$$\frac{dV}{dy} = \frac{K_1 BV}{\gamma} + \frac{Bg}{yV} \tag{7}$$

$$\frac{d\gamma}{dy} = -\frac{K_2 B}{\gamma} + \frac{gB}{V^2 y\gamma}\left(1 - \frac{V^2}{gr}\right) \tag{8}$$

It is noted that these equations can not be evaluated when $\gamma = 0$.

First derivation.- In order to obtain a closed form solution for flight-path angle from equation (8), the following analysis is made. Equation (8) may be written in the form

$$\gamma \, d\gamma = -K_2 B \, dy + \frac{gB}{V^2}\left(1 - \frac{V^2}{gr}\right)\frac{dy}{y} \tag{9}$$

From the mean-value theorems for integrals, the second term on the right can be written as

$$K_3 \int_y^{Y_d} \frac{dy}{y}$$

where K_3 lies between the greatest and the least values of $\frac{gB}{V_1{}^2}\left(1 - \frac{V_1{}^2}{gr}\right)$, or may possibly equal one of them. Since V_1 is actually a variable in this expression, the problem here is that of velocity selection.

Integrating equation (9) gives (for constant V_1)

$$\frac{1}{2}\left(\gamma_d{}^2 - \gamma^2\right) = -K_2 B\left(y_d - y\right) + K_3 \log_e \frac{y_d}{y}$$

But $\log_e y = -\frac{h}{B}$; thus

$$\gamma^2 = \gamma_d{}^2 + 2K_2 B\left(y_d - y\right) - \frac{2K_3}{B}\left(h_d - h\right)$$

Substituting $\Delta h = h_d - h$ and recalling that V_1 is regarded as a constant value of velocity gives

$$\gamma = \pm\left[\gamma_d{}^2 + 2K_2 B\left(y_d - y\right) - 2\frac{g}{V_1{}^2}\Delta h\left(1 - \frac{V_1{}^2}{gr}\right)\right]^{1/2} \tag{10}$$

where, for the conditions chosen, γ will take the sign of Δh.

Second derivation.- Another approach to a closed form solution of equation (8) is to derive a successive approximation procedure. Since velocity is not constant and a closed form solution of equation (7) does not exist, an approximate solution of this equation is to be obtained. A first approximation of flight-path angle obtained from equation (8) is used in equation (7) to yield an approximate solution for velocity. The velocity approximation thus obtained is substituted in equation (8) to yield a second approximation for flight-path angle.

Equation (8) may be written in the form

$$\gamma \frac{d\gamma}{dy} = -K_2 B + \frac{d\epsilon}{dy} \tag{11}$$

where

$$\frac{d\epsilon}{dy} = \frac{gB}{yV^2}\left(1 - \frac{V^2}{gr}\right)$$

It is found from numerical calculations that the second term on the right of equation (7) is usually small in comparison with the first term. By neglecting this term, equation (7) becomes

$$\frac{dV}{dy} = \frac{K_1 BV}{\gamma} \tag{12}$$

The procedure to be followed now is:

    (1) Solve equation (11) for a first approximation of γ

    (2) Use this value of γ in equation (12) to obtain a velocity approximation

    (3) Substitute this value of V in equation (11) to obtain a second approximation for γ

The first approximation of flight-path angle is obtained by integrating equation (11) under the assumption that $d\epsilon/dy = 0$. Hence,

$$\gamma_1{}^2 = \gamma_d{}^2 + 2K_2 B\left(y_d - y\right) \tag{13}$$

A real value of γ_1 is desired under all conditions; hence, in determining γ_1 the absolute value of the expression in equation (13) is taken. The angle γ_1 takes the sign of Δh.

The substitution of γ_1 as defined by equation (13) for γ in equation (12) yields:

$$\frac{dV}{dy} = \frac{K_1 BV}{\left[\gamma_d{}^2 + 2K_2 B\left(y_d - y\right)\right]^{1/2}}$$

This equation may be integrated with the aid of the following substitutions:

$$c = \gamma_d{}^2 + 2K_2 By_d$$

$$z = 2K_2By$$

Thus

$$\frac{dV}{V} = \frac{K_1}{2K_2} \frac{dz}{(c - z)^{1/2}} = \frac{C_D}{2C_L} \frac{dz}{(c - z)^{1/2}}$$

Integrating yields

$$\log_e \frac{V_d}{V} = -\frac{C_D}{C_L}\left[(c - z_d)^{1/2} - (c - z)^{1/2}\right]$$

Since, by definition,

$$z_d = 2K_2By_d$$

it follows that

$$c - z_d = \gamma_d{}^2$$

$$c - z = \gamma_1{}^2$$

$\left(\text{Again it should be noted that } \gamma_1 = \pm\sqrt{|c - z|}.\right)$ Hence,

$$\log_e \frac{V_d}{V} = \frac{C_D}{C_L}\left(\gamma_d - \gamma_1\right)$$

or

$$V = V_d e^{\frac{C_D}{C_L}\left(\gamma_d - \gamma_1\right)} = V_d e^{\frac{C_D}{C_L}\left[\gamma_d - (c-z)^{1/2}\right]} \tag{14}$$

where V_d and γ_d, the desired end conditions, are known.

This solution for V is then used in equation (11) to yield a second approximation for γ. By using equation (14), the second term of equation (11) may be written in the form

$$\frac{d\epsilon}{dy} = -\frac{B}{ry} + \frac{gB}{yV_d^2} e^{\frac{2C_D}{C_L}\left[(c-z)^{1/2}-\gamma_d\right]} \tag{15}$$

Integrating gives

$$\epsilon = -\frac{B}{r}\log_e y\Big|_y^{y_d} + \int \frac{gB}{V_d^2} e^{\frac{2C_D}{C_L}\left[(c-z)^{1/2}-\gamma_d\right]} \frac{dy}{y} \tag{16}$$

Since $\dfrac{dy}{y} = \dfrac{dz}{z}$, the second term of this equation becomes

$$\int Ae^{\frac{2C_D}{C_L}(c-z)^{1/2}} \frac{dz}{z}$$

where

$$A = \frac{gB}{V_d^2} e^{\frac{-2C_D}{C_L}\gamma_d}$$

This integral may be evaluated as follows: Let $x = (c - z)^{1/2}$ where c is a constant. Then

$$z = c - x^2$$

$$dz = -2x\,dx$$

$$\frac{dz}{z} = -\frac{2x\,dx}{c - x^2}$$

Hence the integral becomes

$$\int Ae^{\frac{2C_D}{C_L}x}\left(\frac{-2x\,dx}{c - x^2}\right)$$

or

$$\int \frac{2Axe^{\frac{2C_D}{C_L}x}}{x^2 - c} \, dx$$

By partial fractions,

$$\frac{2Ax}{\left(x + \sqrt{c}\right)\left(x - \sqrt{c}\right)} = \frac{A}{x + \sqrt{c}} + \frac{A}{x - \sqrt{c}}$$

Thus the integral may now be written

$$\int \frac{A}{x + \sqrt{c}} \, e^{\frac{2C_D}{C_L}x} \, dx + \int \frac{A}{x - \sqrt{c}} \, e^{\frac{2C_D}{C_L}x} \, dx$$

In order to integrate this expression, let

$$w_1 = x + \sqrt{c}$$

$$w_2 = x - \sqrt{c}$$

Then

$$w_{1,d} = x_d + \sqrt{c} = \gamma_d + \sqrt{c}$$

$$w_{2,d} = x_d - \sqrt{c} = \gamma_d - \sqrt{c}$$

The expression to be integrated becomes

$$\int \frac{A}{w_1} \, e^{\frac{2C_D}{C_L}\left(w_1 - \sqrt{c}\right)} \, dw_1 + \int \frac{A}{w_2} \, e^{\frac{2C_D}{C_L}\left(w_2 + \sqrt{c}\right)} \, dw_2$$

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Integrating yields

$$Ae^{\frac{-2C_D}{C_L}\sqrt{c}}\left[\log_e w_1 + \sum_{n=1}^{\infty} \frac{\left(\frac{2C_D}{C_L} w_1\right)^n}{n \cdot n!}\right]\Bigg|_{w_1}^{w_{1,d}} + Ae^{\frac{2C_D}{C_L}\sqrt{c}}\left[\log_e w_2 + \sum_{n=1}^{\infty} \frac{\left(\frac{2C_D}{C_L} w_2\right)^n}{n \cdot n!}\right]\Bigg|_{w_2}^{w_{2,d}}$$

Truncating the series at $n = 1$ gives

$$Ae^{\frac{-2C_D}{C_L}\sqrt{c}}\left[\log_e \frac{w_{1,d}}{w_1} + \frac{2C_D}{C_L}\left(w_{1,d} - w_1\right)\right] + Ae^{\frac{2C_D}{C_L}\sqrt{c}}\left[\log_e \frac{w_{2,d}}{w_2} + \frac{2C_D}{C_L}\left(w_{2,d} - w_2\right)\right]$$

Since $w_{1,d} - w_1 = w_{2,d} - w_2$ and $\dfrac{w_1}{w_{1,d}}\dfrac{y_d}{y} = \dfrac{w_{2,d}}{w_2}$, this expression may be written in the form:

$$A\left\{e^{\frac{-2C_D}{C_L}\sqrt{c}}\log_e \frac{w_{1,d}}{w_1} + e^{\frac{2C_D}{C_L}\sqrt{c}}\log_e \frac{w_1}{w_{1,d}}\frac{y_d}{y} + \frac{2C_D}{C_L}\left(w_{1,d} - w_1\right)\left[e^{\left(\frac{2C_D}{C_L}\sqrt{c}\right)} + e^{\left(\frac{-2C_D}{C_L}\sqrt{c}\right)}\right]\right\}$$

or

$$A\left[2\sinh\frac{2C_D}{C_L}\sqrt{c}\,\log_e \frac{w_1}{w_{1,d}} + \frac{C_D}{C_L}\left(w_{1,d} - w_1\right)\left(4\cosh\frac{2C_D}{C_L}\sqrt{c}\right) + e^{\frac{2C_D}{C_L}\sqrt{c}}\log_e \frac{y_d}{y}\right]$$

Hence, equation (16) becomes:

$$\epsilon = -\frac{B}{r}\log_e \frac{y_d}{y} + \frac{gB}{V_d^2}e^{\frac{-2C_D}{C_L}\gamma_d}\left[\left(2\sinh\frac{2C_D}{C_L}\sqrt{c}\right)\left(\log_e \frac{w_1}{w_{1,d}}\right)\right.$$

$$\left. + \frac{4C_D}{C_L}\left(w_{1,d} - w_1\right)\cosh\frac{2C_D}{C_L}\sqrt{c} + e^{\frac{2C_D}{C_L}\sqrt{c}}\log_e \frac{y_d}{y}\right] \qquad (17)$$

where $\sqrt{c}$ takes the sign of Δh.

It should be noted that equation (17) can be used to evaluate ϵ under all conditions except zero lift by assuming

$$\sqrt{c} = \sqrt{|c|}$$

$$\sinh \frac{2c_D}{c_L} \sqrt{c} = \sinh \left| \frac{2c_D}{c_L} \sqrt{c} \right|$$

$$\cosh \frac{2c_D}{c_L} \sqrt{c} = \cosh \left| \frac{2c_D}{c_L} \sqrt{c} \right|$$

Now that the integral of $\frac{d\epsilon}{dy}$ has been obtained, equation (11) can be integrated thus:

$$\frac{1}{2}\left(\gamma_d{}^2 - \gamma^2\right) = -K_2\left(y_d - y\right) \pm \epsilon$$

Solving for γ

$$\gamma = \pm\left[\gamma_d{}^2 + 2K_2B\left(y_d - y\right) \pm 2\epsilon\right]^{1/2}$$

or

$$\gamma = \left(\gamma_1{}^2 \pm 2\epsilon\right)^{1/2} \tag{18}$$

The positive sign is chosen when ϵ (obtained by eq. (17)) and Δh have like signs, and the negative sign when ϵ and Δh have unlike signs. The angle γ takes the sign of Δh.

Procedure

As a basis for evaluating the equations derived in the preceding sections, the equations of motion (eqs. (1) to (3)) were solved in negative time, by using a digital computer, to provide trajectories with end conditions $\left(V_d, \ \gamma_d, \ h_d\right)$ specified for this study. These trajectories are hereinafter referred to as exact solutions. Exact altitude and velocity values (h, V) at various

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positions along these trajectories were used to calculate the approximate value of flight-path angle.

In order to facilitate the discussion, the approximations obtained by the various equations shall be referred to as methods I to IV. In method I, equation (13) is evaluated to provide a rapid first approximation for γ. This equation becomes a part of the approximations used in succeeding methods. Method II evaluates equation (10) in which the present velocity is chosen for the constant-velocity value. In method III, which also evaluates equation (10), the velocity as obtained from equation (14) is used for the constant-velocity value. Method IV evaluates equation (18). An approximate solution for velocity is used in the development of the closed form equation for flight-path angle. All four methods were programed on a digital computer. It is to be noted that the program logic for method IV was considerably more complicated than that for the other methods.

RESULTS AND DISCUSSION

Results of the study are presented as time histories of the exact and the calculated values of flight-path angle. Time histories of the exact velocity and altitude values used in the approximate equations are included for reference.

For the most part, the results of the present study are given for a high-drag low-lift vehicle having a W/S ratio of 60 lb/ft^2 and a lift-drag ratio of ±0.5. Limited results are shown for this vehicle where the lift-drag ratio was trimmed at 0.2. Results are also presented for a high-lift reentry vehicle which has a W/S ratio of 27.88 lb/ft^2 and is trimmed at a lift-drag ratio of 1.5.

Desired conditions were selected at three altitudes: 200,000 feet, 250,000 feet, and 300,000 feet. For the high-drag low-lift vehicle, these altitudes are altitudes where precise path control is of primary concern. An altitude of 200,000 feet is the (approximate) minimum altitude allowable during the initial dive into the atmosphere where overdeceleration (greater than 8g) of the vehicle's passenger will not occur. An altitude of 250,000 feet is considered the appropriate altitude at which to level off (after bleeding off excessive velocity at a lower altitude) in order to obtain certain desired ranges. At 300,000 feet, the density is very low; in many studies, this altitude is regarded as the upper boundary of the usable atmosphere.

Hence, for desired end conditions for the high-drag low-lift vehicle at h_d = 200,000 feet, velocity values ranging from 20,000 ft/sec to 40,000 ft/sec with γ_d = 0 were chosen. For h_d = 250,000 feet, velocities were varied from 26,000 ft/sec to 30,000 ft/sec while γ_d varied from 0° to 1°. For h_d = 300,000 feet, flight-path angles of 1° and 2° were selected for velocities of 25,000 ft/sec and 26,000 ft/sec. Results for the high-lift reentry vehicle were calculated for velocities of 26,000 ft/sec, 20,000 ft/sec, and 14,000 ft/sec for h_d = 200,000 feet and γ_d = 0°.

Other cases were investigated wherein satisfactory correlation between the exact and approximate values were derived; however, cases where tolerable g values were exceeded have been omitted.

High-Drag Low-Lift Vehicle

$L/D = \pm 0.5$.- In figure 1, results are shown for cases where the desired end conditions are level flight at an altitude of 200,000 feet for four specified velocities. At V_d = 40,000 ft/sec, 30,000 ft/sec, and 26,000 ft/sec for $C_L = \pm 0.5$, excellent agreement with the exact values is obtained by methods II and IV. As noted in the figure, a low value of γ is approximated by method I for these velocities. At suborbital velocity $(V_d$ = 20,000 ft/sec$)$, the best approximations are obtained by methods II and III. As shown in figures 1(d) and 1(e), values obtained by methods I and IV are considerably higher than the exact values for C_L = 0.5, whereas for C_L = -0.5, these two methods give low values.

Cases were calculated for desired altitude of 250,000 feet for V_d = 30,000 ft/sec and 26,000 ft/sec for $\gamma_d = 1^O$ as well as $\gamma_d = 0^O$ for both positive and negative values of C_L. These results are shown in figure 2. For $\gamma_d = 0^O$, methods II and III give consistently good results for both positive and negative lift; excellent results are obtained by method IV for positive lift, as noted in figures 2(a) and 2(c). For $\gamma_d = 1^O$ (figs. 2(e) to 2(h)), the most accurate results are obtained by using methods II and IV. Method I gave very poor approximations for the cases where V_d = 30,000 ft/sec; however, all methods were satisfactory for V_d = 26,000 ft/sec.

Figure 3 shows results for the cases where h_d = 300,000 feet for V_d = 26,000 ft/sec and 25,000 ft/sec. For V_d = 26,000 ft/sec and $\gamma_d = 1^O$, excellent predictions of flight-path angle are obtained by methods III and IV for both positive and negative lift. (See figs. 3(a) and 3(b).)

For $\gamma_d = 2^O$, all methods except method I yield satisfactory results as noted in figures 3(c) and 3(d). For the case where V_d = 25,000 ft/sec and $\gamma_d = 2^O$, excellent agreement with the exact values is obtained by method III and good agreement by method II. It is noted in figures 3(e) and 3(f) that values calculated by methods I and IV are unsatisfactory for both positive and negative lift.

$L/D = \pm 0.2$.- The four approximate equations were evaluated for selected cases where the lift-drag ratio of the reentry vehicle was trimmed to ± 0.2. As illustrated in figures 4(a) and 4(b), all methods provide satisfactory results for either $\gamma_d = 0^O$ or $\gamma_d = 2^O$, for h_d = 200,000 ft, V_d = 26,000 ft/sec, and C_L = 0.2. Shown in figures 4(c) and 4(d) are cases for h_d = 300,000 ft,

$V_d = 25,000$ ft/sec, $\gamma_d = 2°$, $C_L = \pm 0.2$; method III provided excellent results and method II good results for both positive and negative lift. For $h_d = 200,000$ ft, $V_d = 20,000$ ft/sec, and $\gamma_d = 0°$, both methods II and III gave excellent results for $C_L = 0.2$. (See fig. 4(e).)

In a comparison of figures 3(e) and 3(f) and figures 4(c) and 4(d), it can be seen that methods I and IV are unsatisfactory in predicting flight-path angle for suborbital velocities at $h = 300,000$ feet. As noted in figures 4(e), 1(d), and 1(e), this is also true for suborbital velocities at $h = 200,000$ feet.

High-Lift Vehicle

Shown in figure 5 are results calculated for a high-lift reentry vehicle (L/D = 1.5) where desired end conditions were level flight at an altitude of 200,000 feet. It may be seen in figure 5(a) that excellent correlation with the exact values is provided by methods I and IV for $V_d = 26,000$ ft/sec.

Method II provided excellent results also for suborbital velocities, $V_d = 20,000$ ft/sec, and $V_d = 14,000$ ft/sec, as shown in figures 5(b) and 5(c).

CONCLUDING REMARKS

An analysis of the equations of motion for a space vehicle reentering the earth's atmosphere has been made. The objective of the analysis was to develop a closed form solution for flight-path angle that could be used to predict the path parameters over a short time interval. Only acceptable reentry conditions were investigated.

The results of this analysis have shown that the flight-path angle of a reentry vehicle can be approximated with some degree of accuracy by several methods. The approximations are valid in the altitude range between 200,000 feet and 300,000 feet, for velocities ranging from suborbital to hyperbolic, for a high-drag low-lift vehicle. For a high lifting vehicle, the approximations are valid for suborbital to orbital velocities at an altitude of 200,000 feet.

Langley Research Center,
National Aeronautics and Space Administration,
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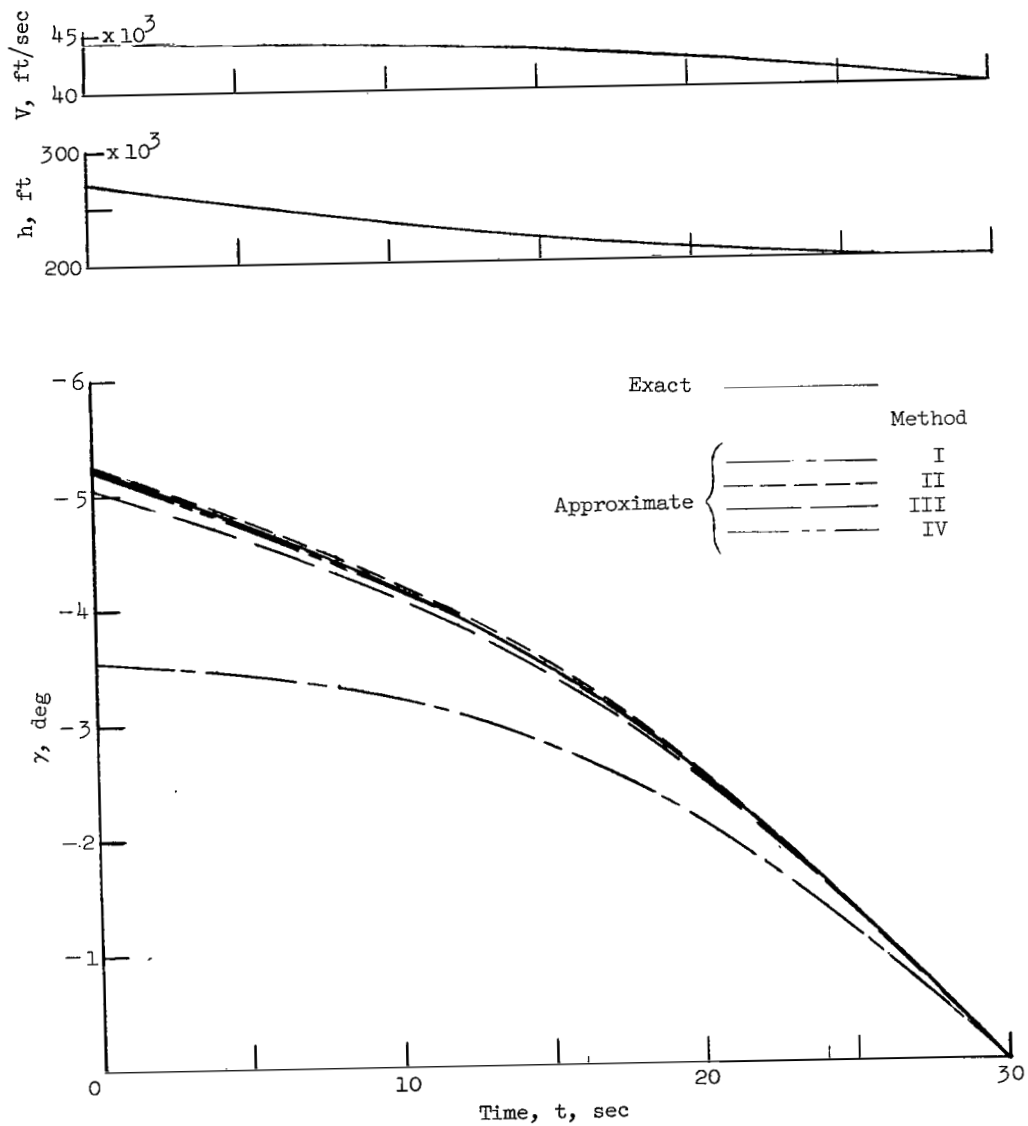
(a) V_d = 40,000 ft/sec; C_L = 0.5.

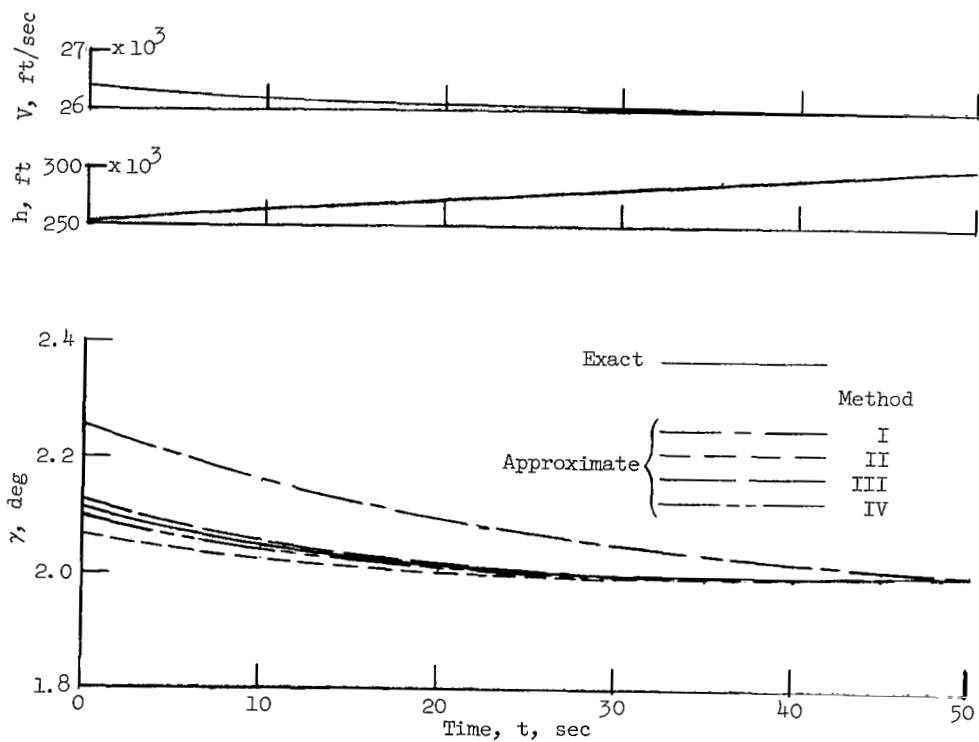Figure 1.- Time history of exact trajectory and calculated values of flight-path angle for desired altitude of 200,000 feet. $\gamma_d = 0^{\circ}$.
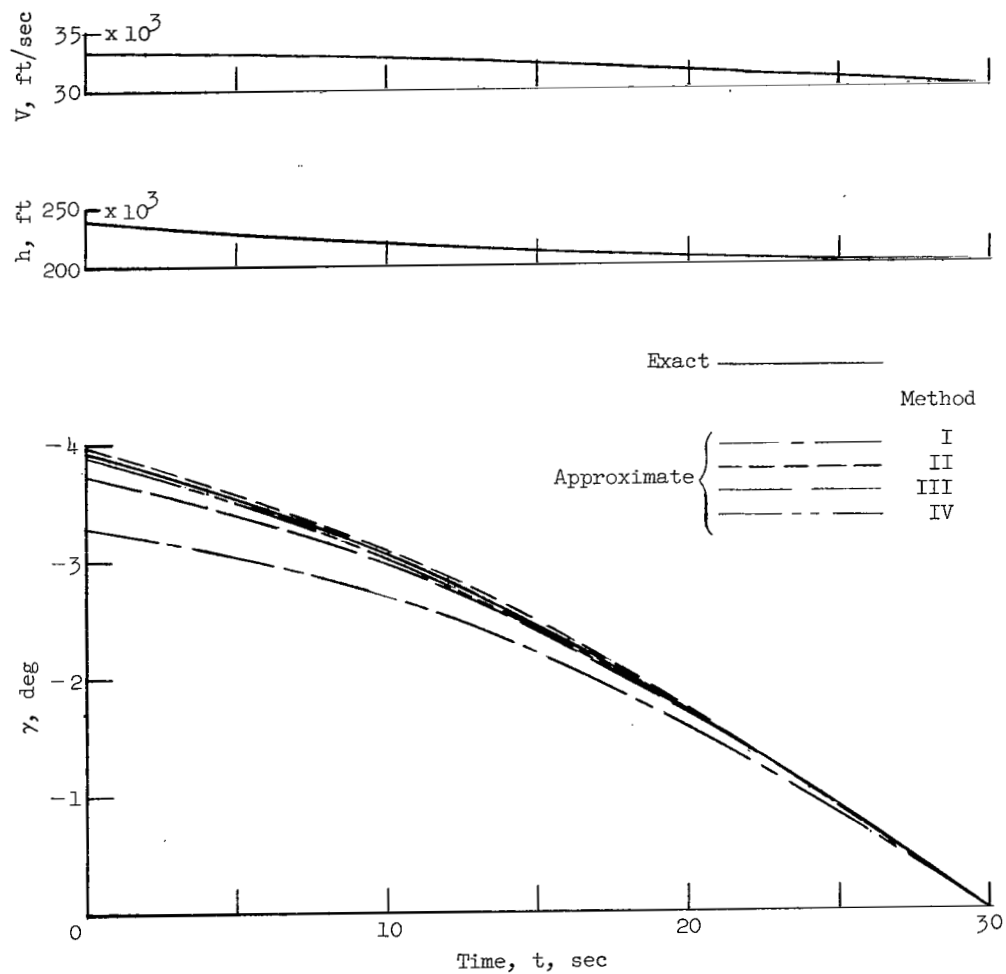
(b) V_d = 30,000 ft/sec; C_L = 0.5.

Figure 1.- Continued.
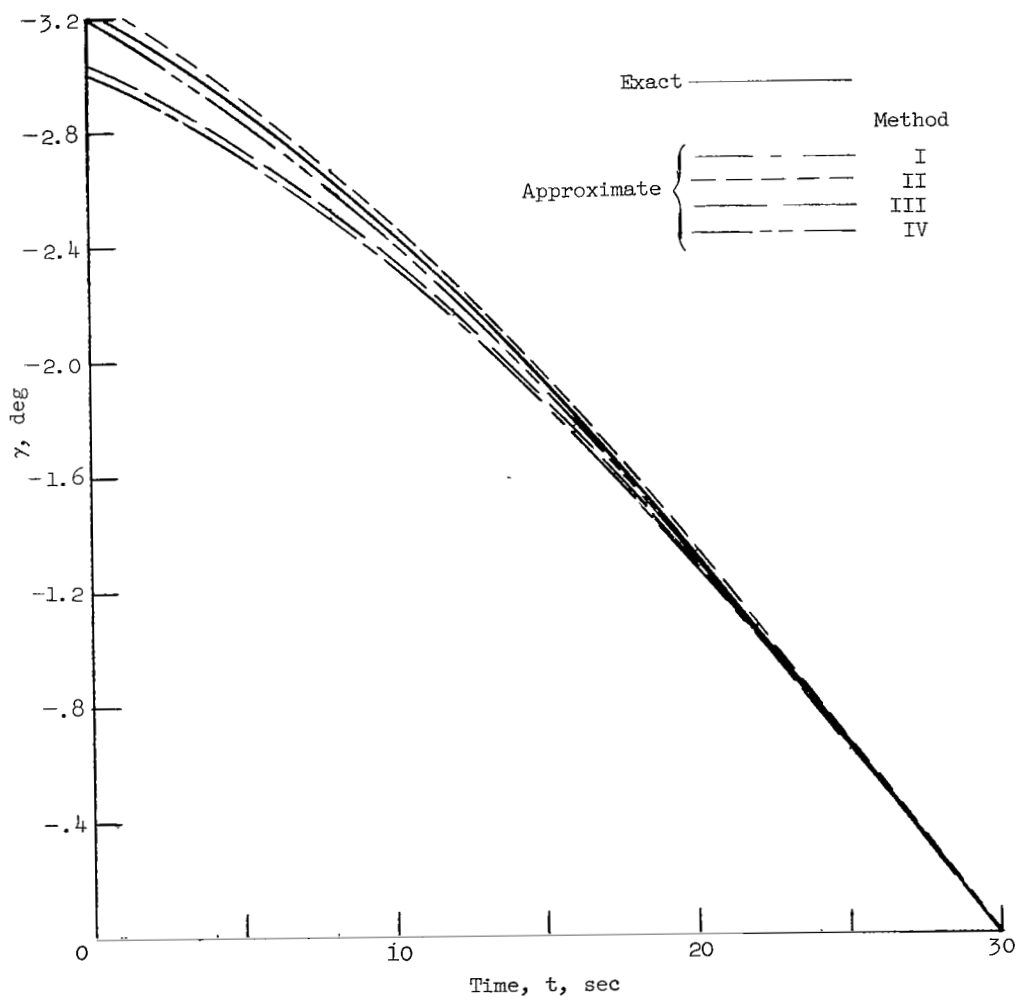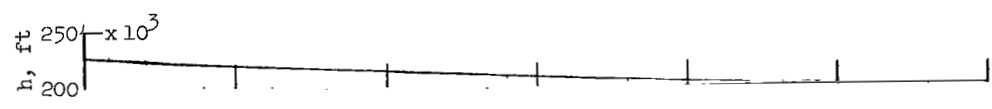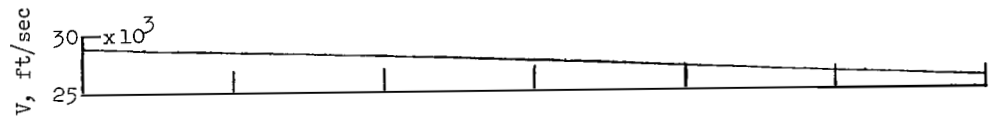
(c) $V_d = 26,000$ ft/sec; $C_L = 0.5$.

Figure 1.- Continued.
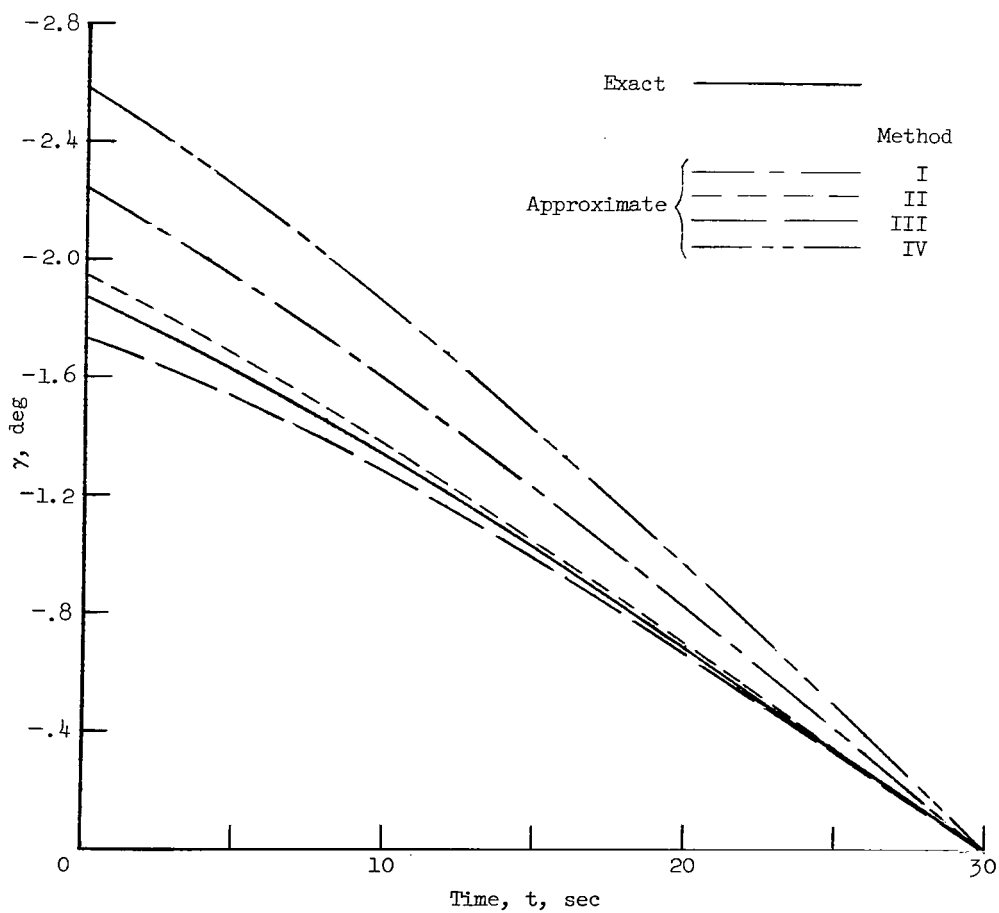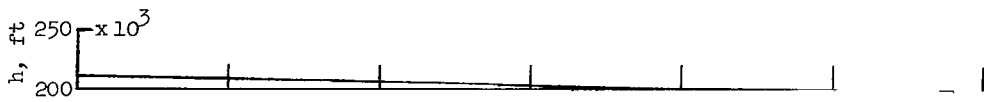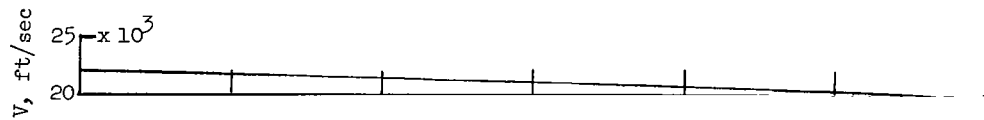
(d) V_d = 20,000 ft/sec; C_L = 0.5.
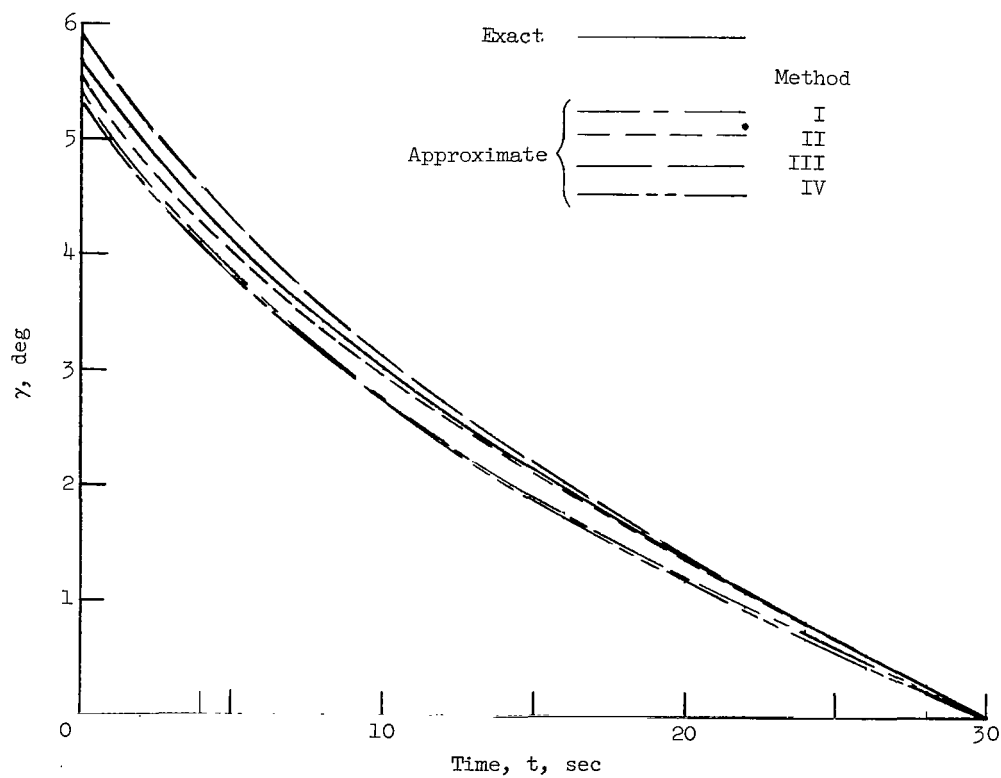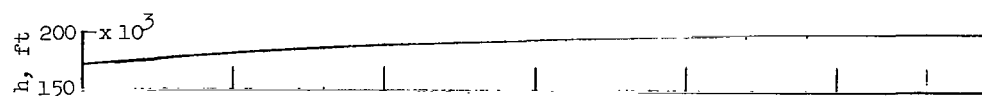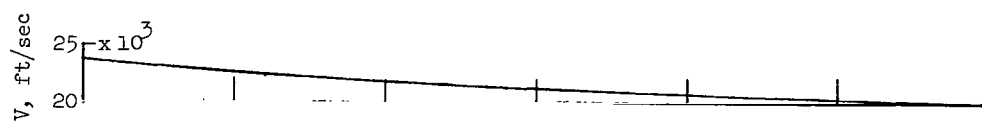
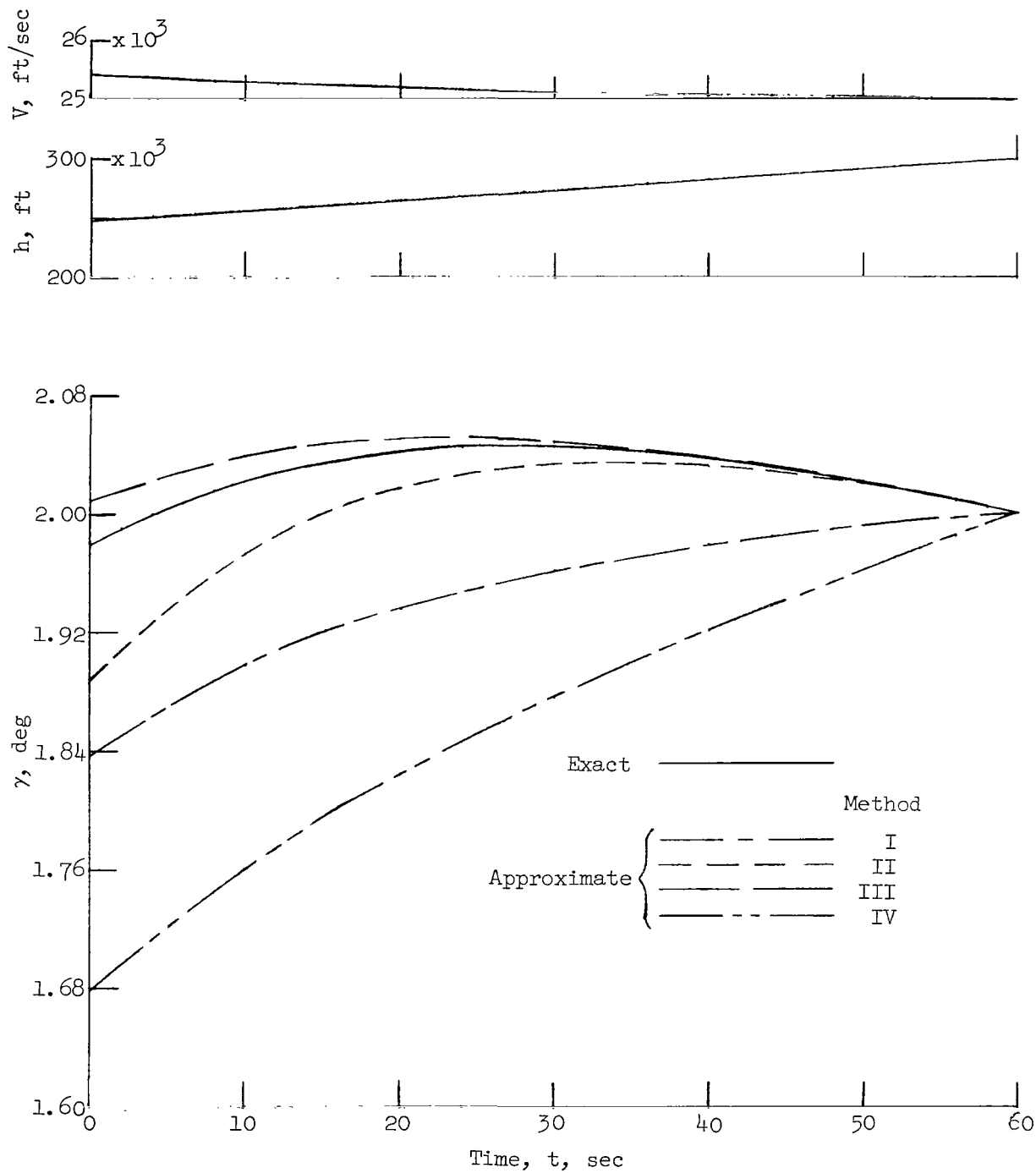Figure 1.- Continued.

(e) $V_d = 20,000$ ft/sec; $C_L = -0.5$.

Figure 1.- Concluded.

(a) V_d = 30,000 ft/sec; γ_d = 0°; C_L = 0.5.

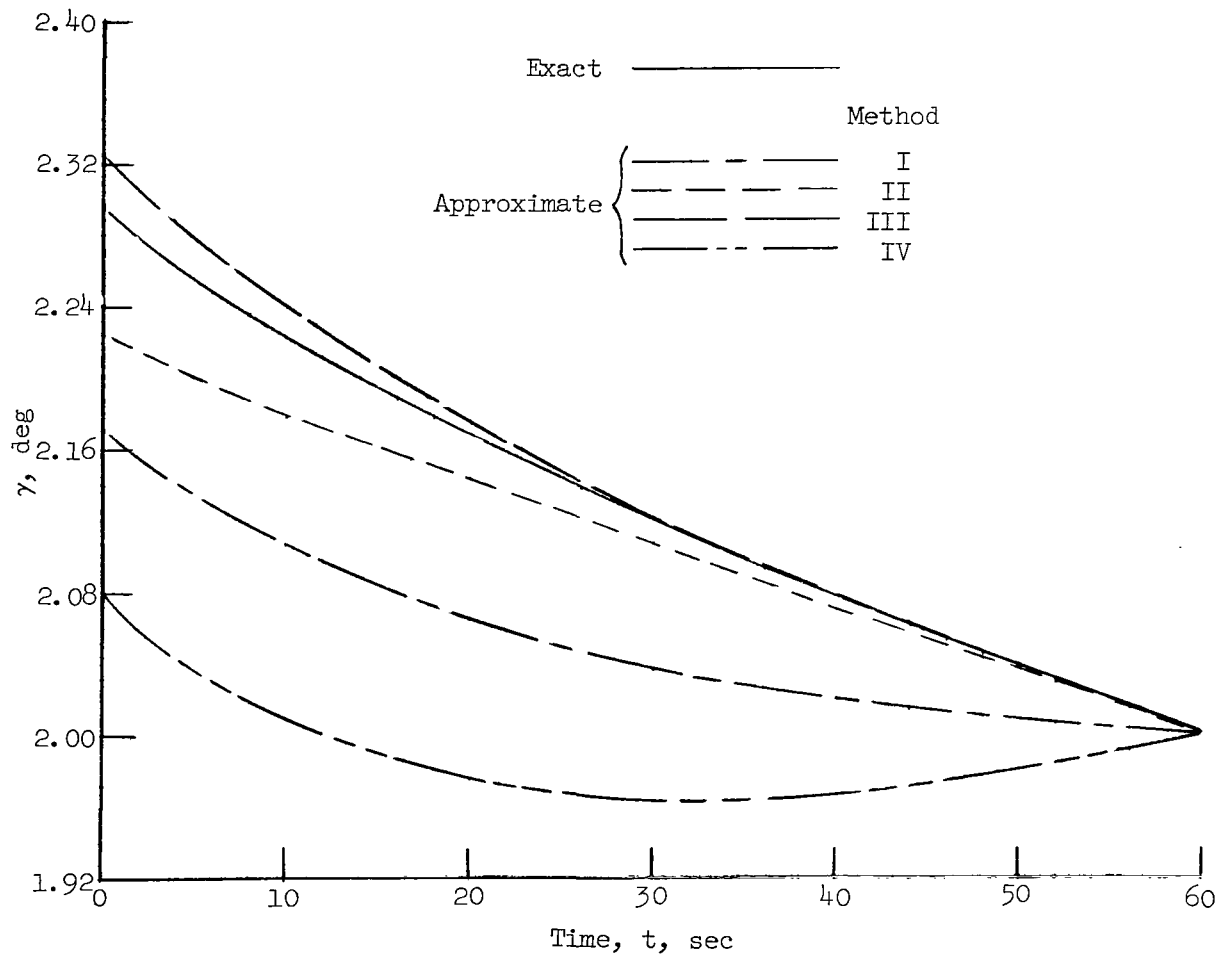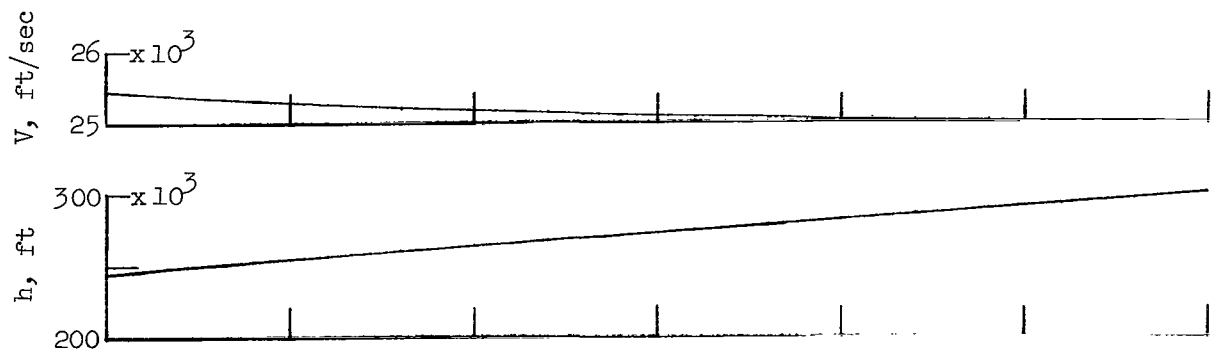Figure 2.- Time history of exact trajectory and calculated values of flight-path angle for desired altitude of 250,000 feet.
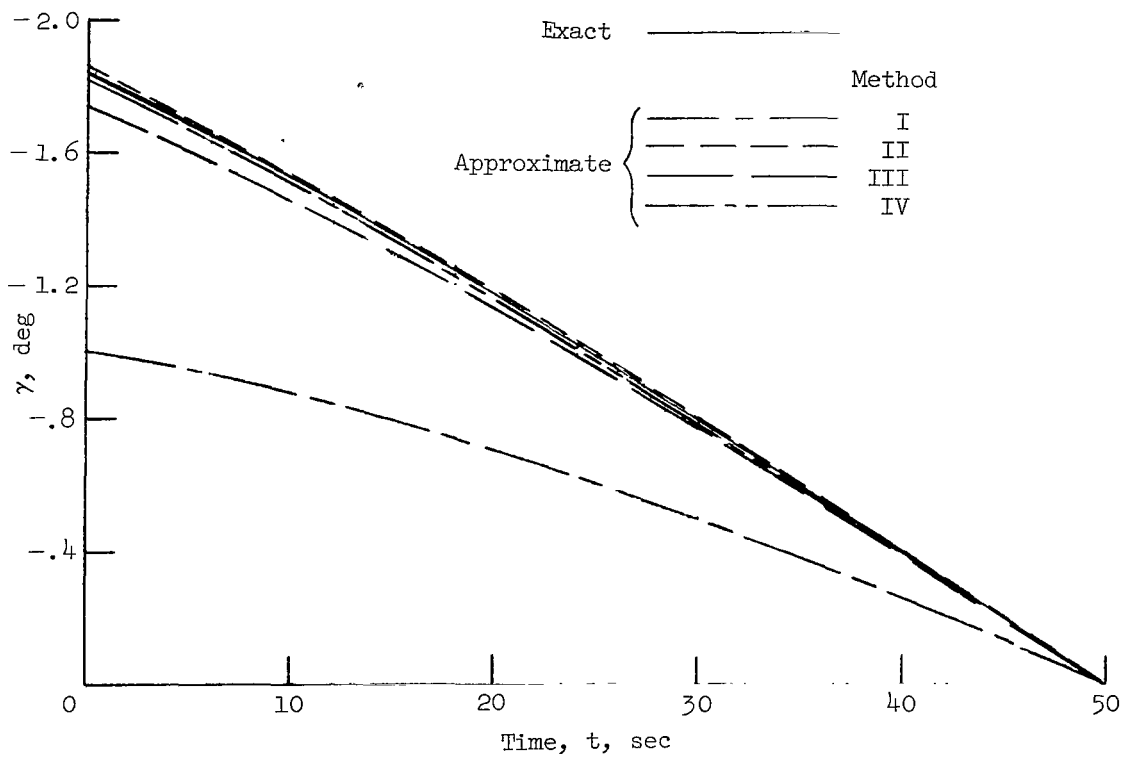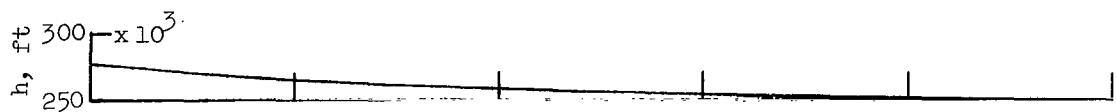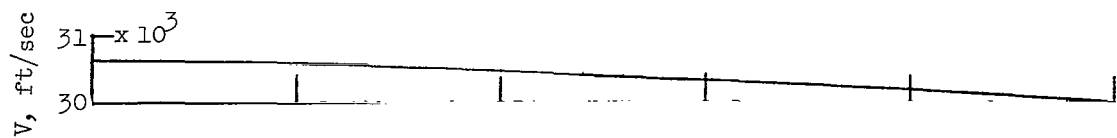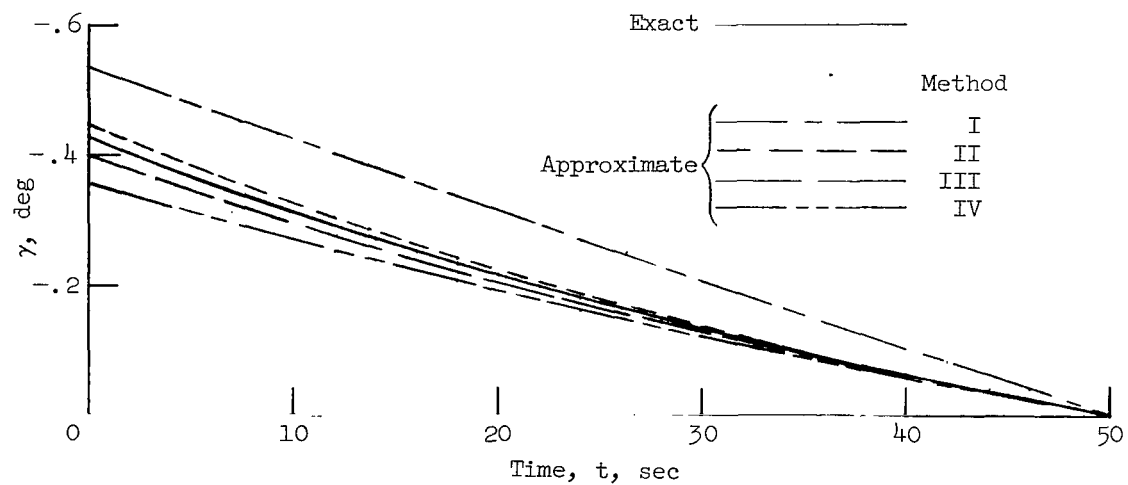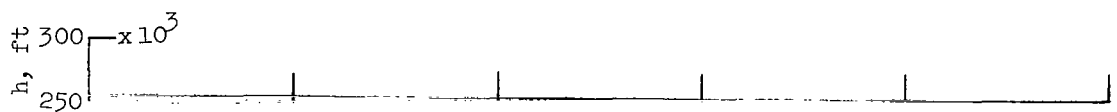
(b) $V_d = 30,000$ ft/sec; $\gamma_d = 0°$; $C_L = -0.5$.

Figure 2.- Continued.

(c) V_d = 26,000 ft/sec; γ_d = 0°; C_L = 0.5.
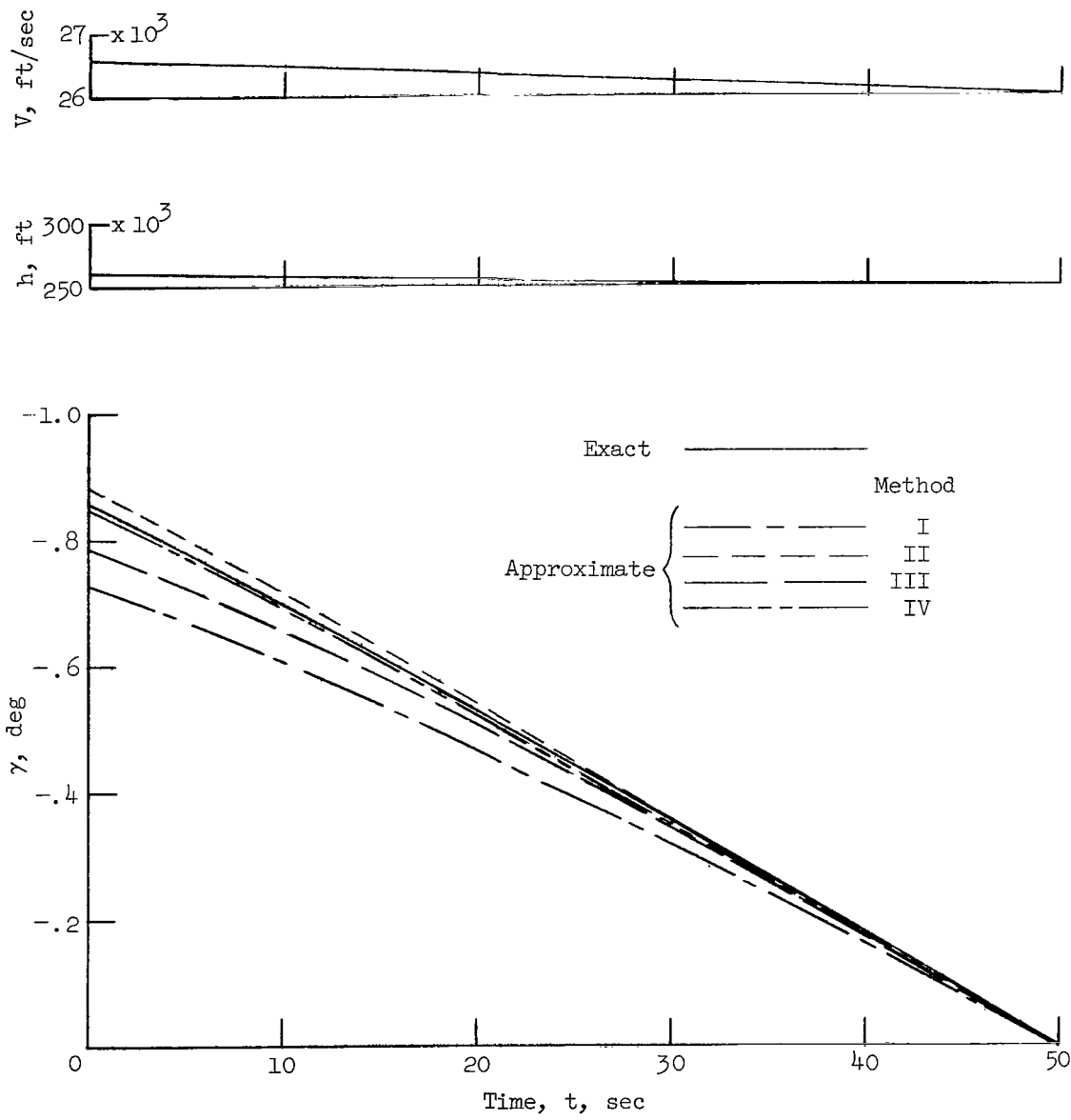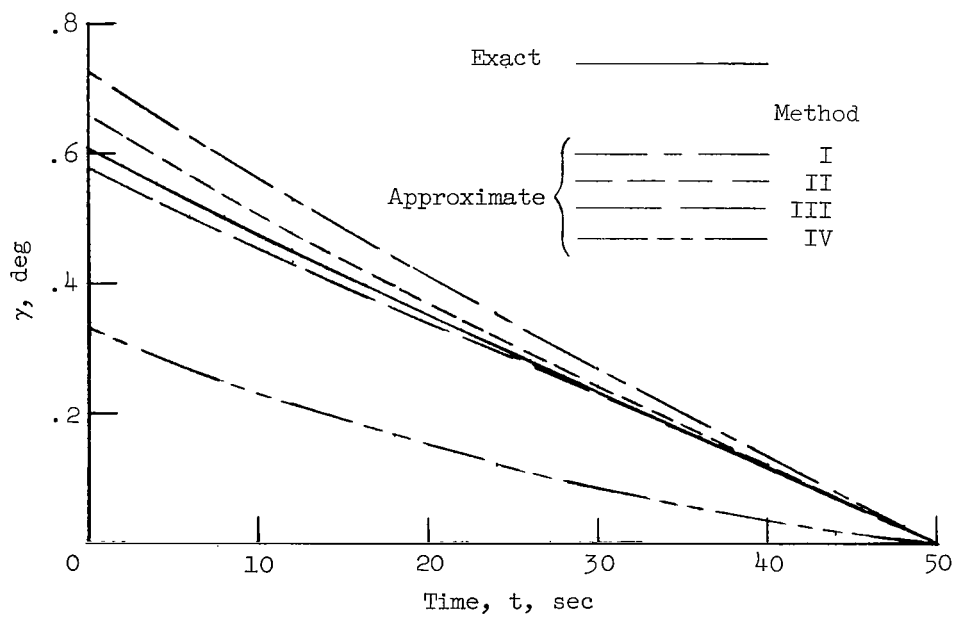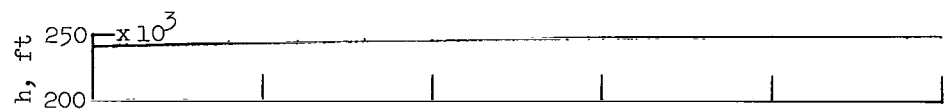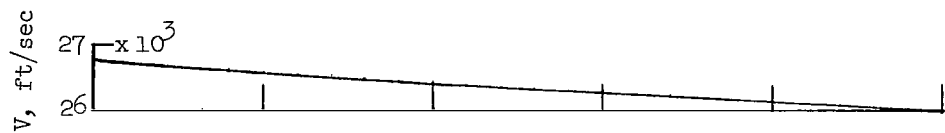
Figure 2.- Continued.

(d) $V_d = 26,000$ ft/sec; $\gamma_d = 0°$; $C_L = -0.5$.

Figure 2.- Continued.

(e) V_d = 30,000 ft/sec; γ_d = 1°; C_L = 0.5.

Figure 2.- Continued.

(f) V_d = 30,000 ft/sec; γ_d = 1°; C_L = -0.5.

Figure 2.- Continued.

(g) V_d = 26,000 ft/sec; γ_d = 1°; C_L = 0.5.
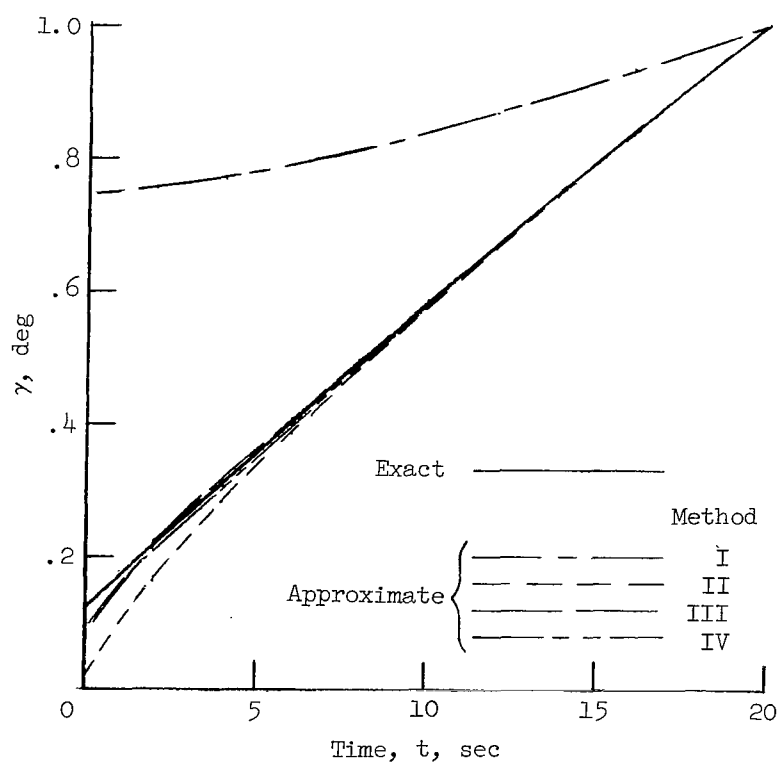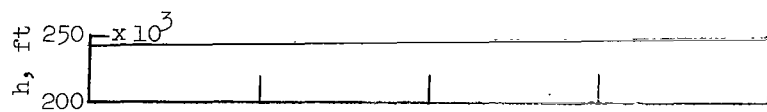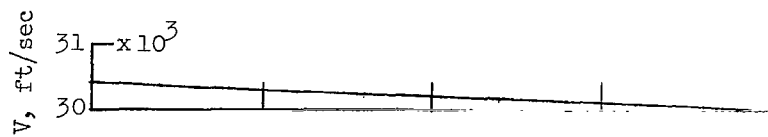
Figure 2.- Continued.

(h) $V_d = 26,000$ ft/sec; $\gamma_d = 1°$; $C_L = -0.5$.

Figure 2.- Concluded.

(a) V_d = 26,000 ft/sec; γ_d = 1°; C_L = 0.5.

Figure 3.- Time history of exact trajectory and calculated values of flight-path angle for desired altitude of 300,000 feet.

(b) V_d = 26,000 ft/sec; γ_d = 1°; C_L = -0.5.

Figure 3.- Continued.
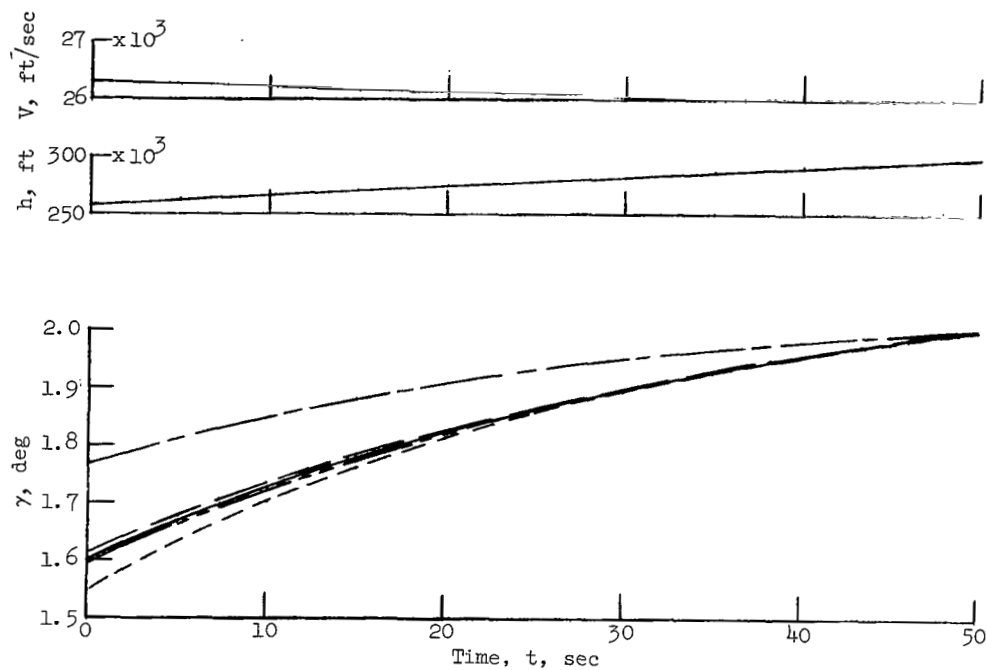
(c) $V_d = 26,000$ ft/sec; $\gamma_d = 2°$; $C_L = 0.5$.

(d) $V_d = 26,000$ ft/sec; $\gamma_d = 2°$; $C_L = -0.5$.

Figure 3.- Continued.

(e) $V_d = 25,000$ ft/sec; $\gamma_d = 2^0$; $C_L = 0.5$.

Figure 3.- Continued.

(f) V_d = 25,000 ft/sec; γ_d = 2°; C_L = -0.5.

Figure 3.- Concluded.
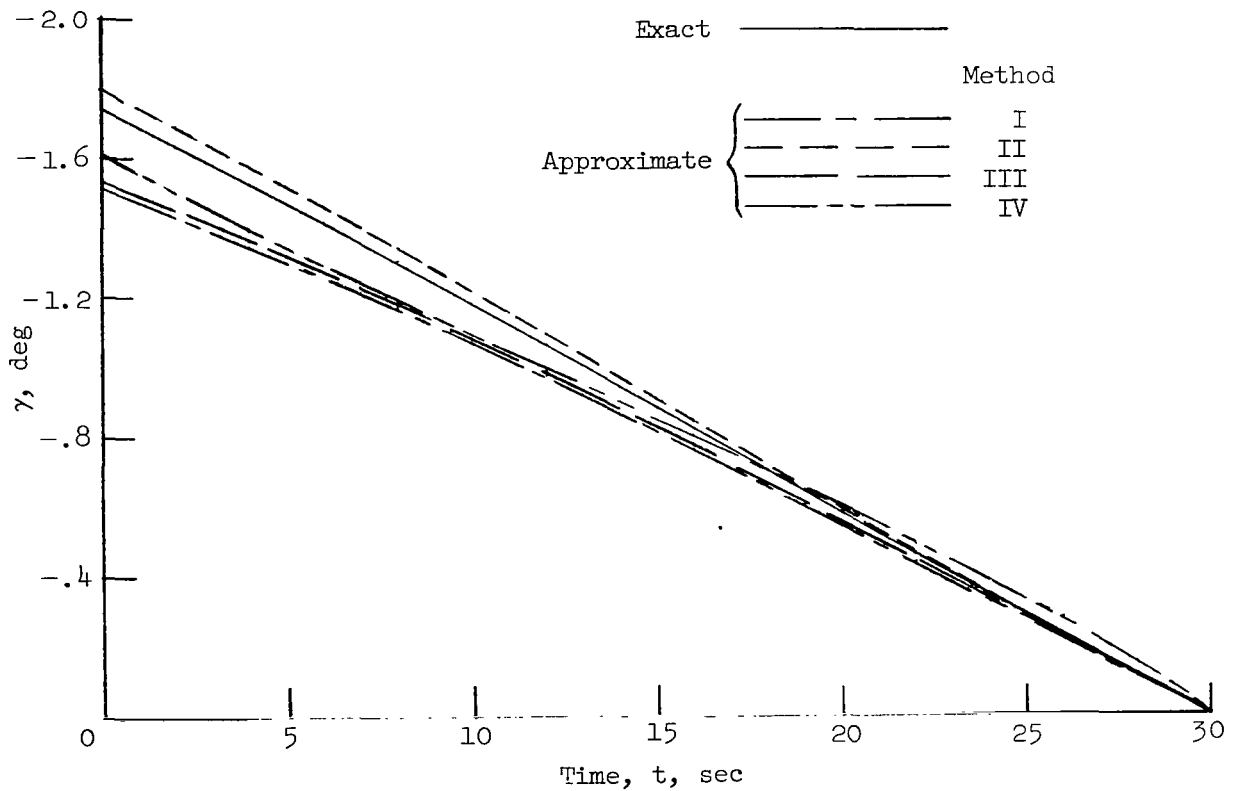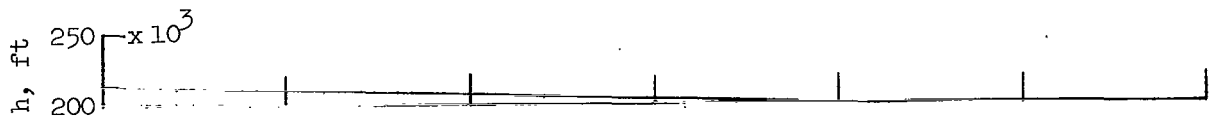
(a) h_d = 200,000 ft; V_d = 26,000 ft/sec; γ_d = 0°; C_L = 0.2.

Figure 4.- Time history of exact trajectory and calculated values of flight-path angle.
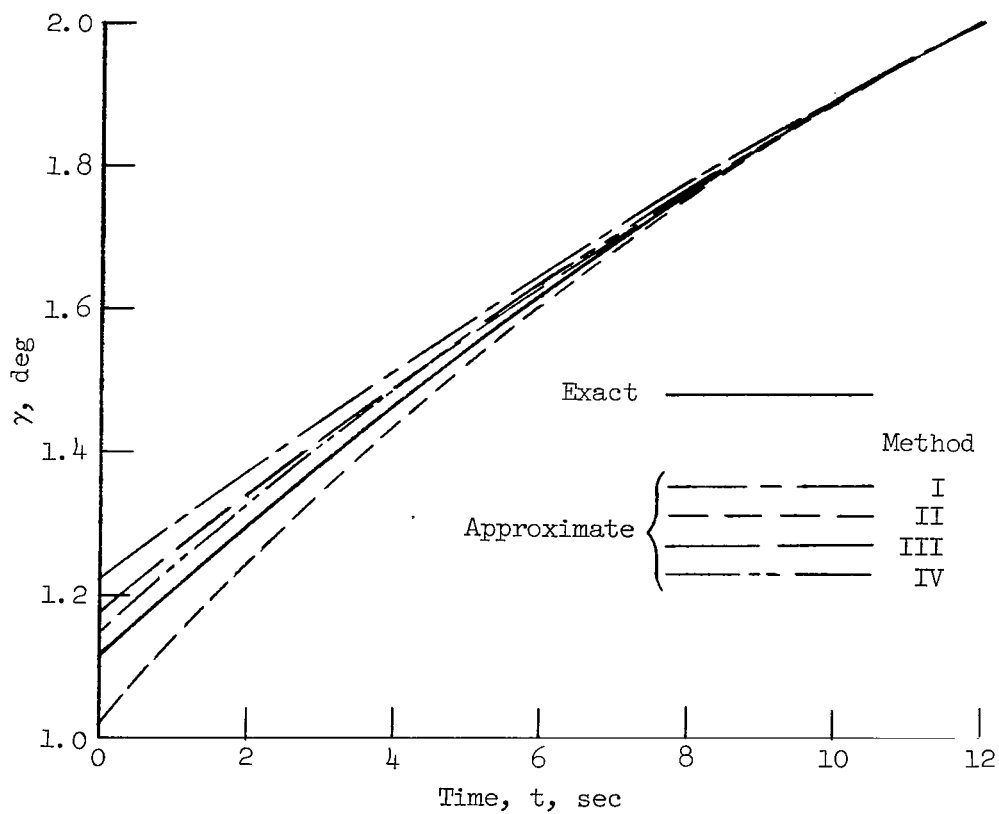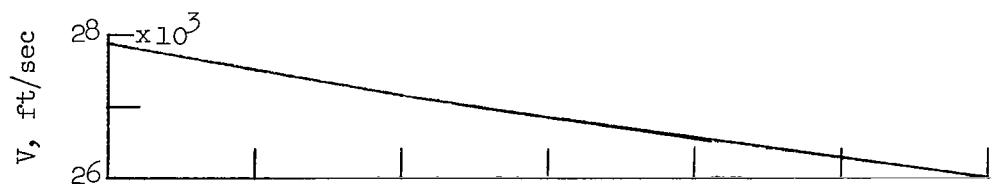
(b) h_d = 200,000 ft; V_d = 26,000 ft/sec; γ_d = 2°; C_L = 0.2.

Figure 4.- Continued.

(c) h_d = 300,000 ft; V_d = 25,000 ft/sec; γ_d = 2°; C_L = 0.2.

Figure 4.- Continued.

(d) h_d = 300,000 ft; V_d = 25,000 ft/sec; γ_d = 2°; C_L = -0.2.
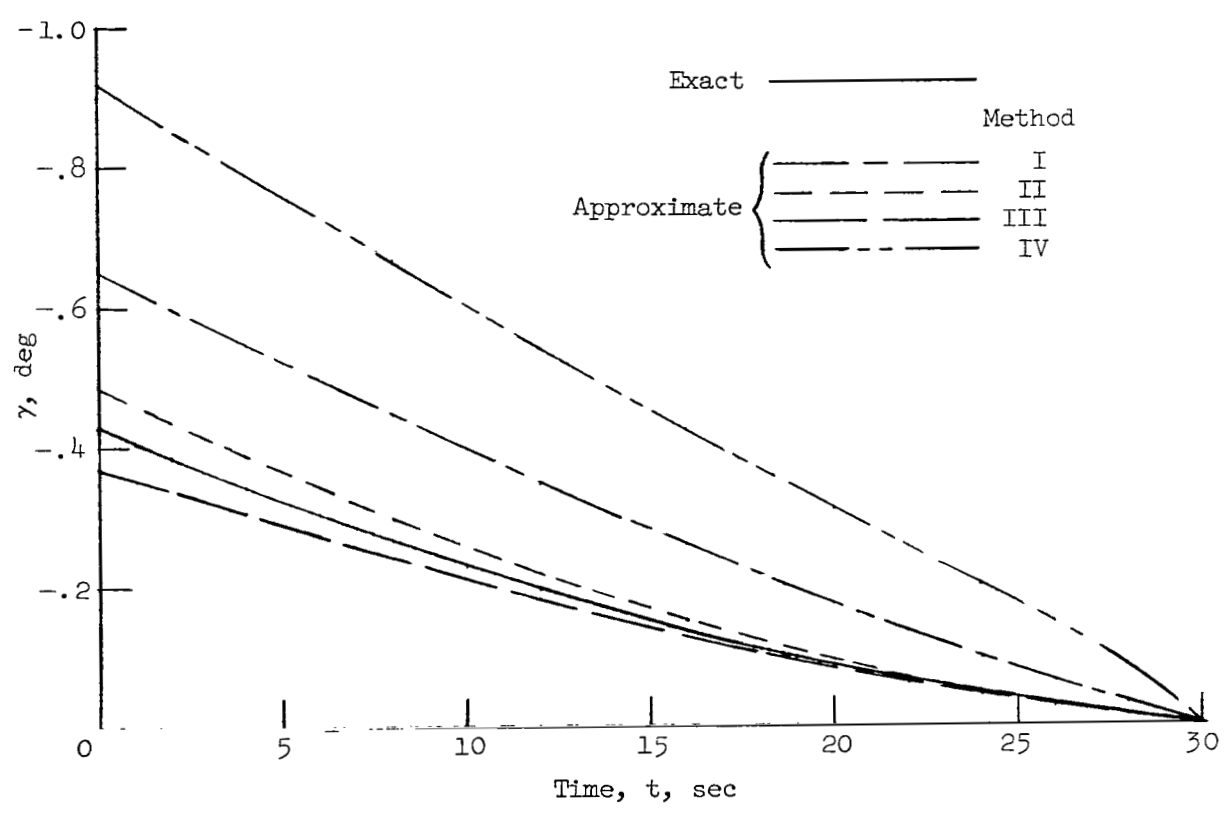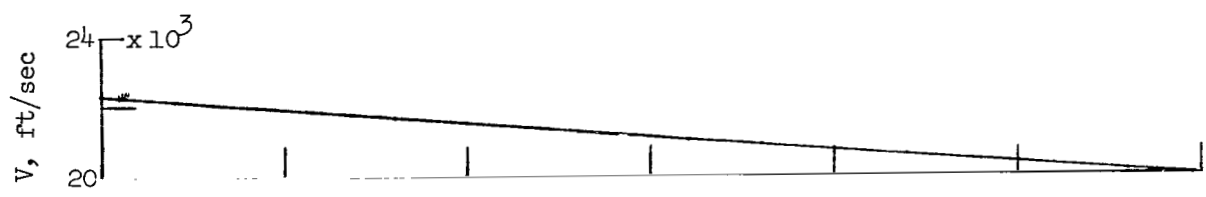
Figure 4.- Continued.

(e) h_d = 200,000 ft; V_d = 20,000 ft/sec; γ_d = 0°; C_L = 0.2.
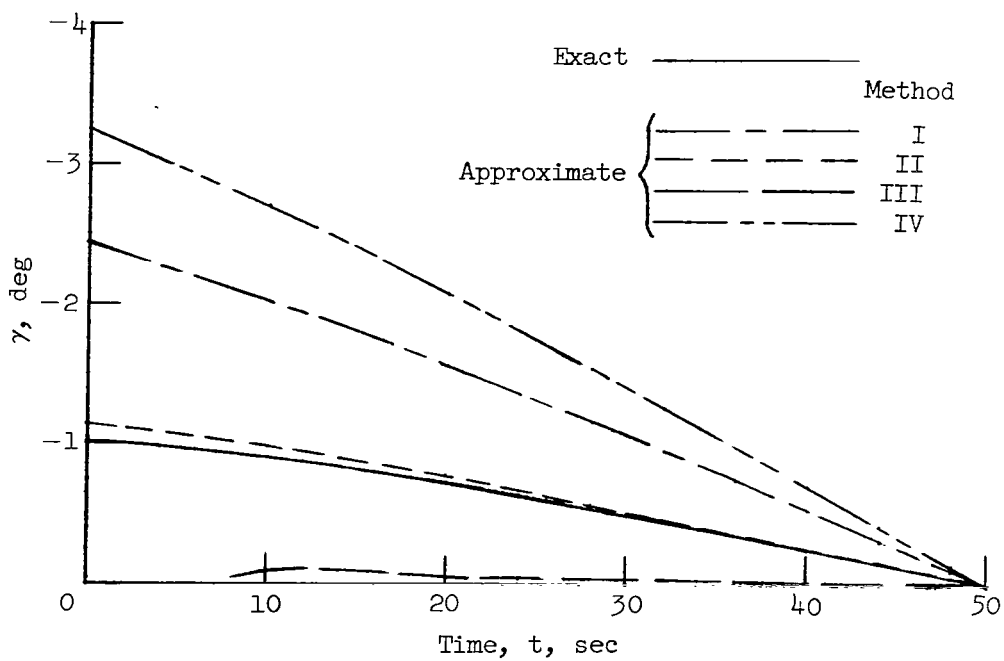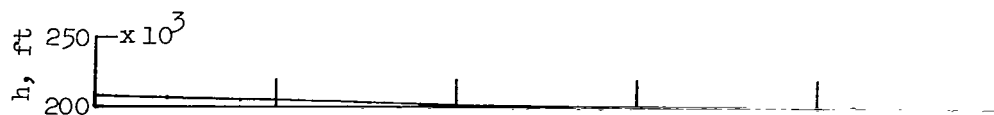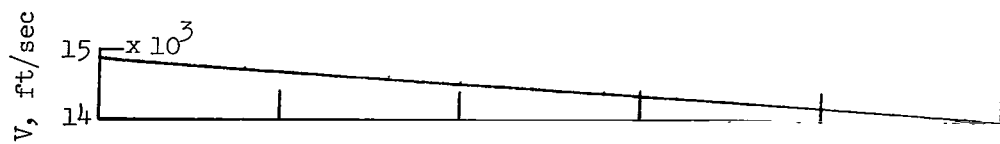
Figure 4.- Concluded.
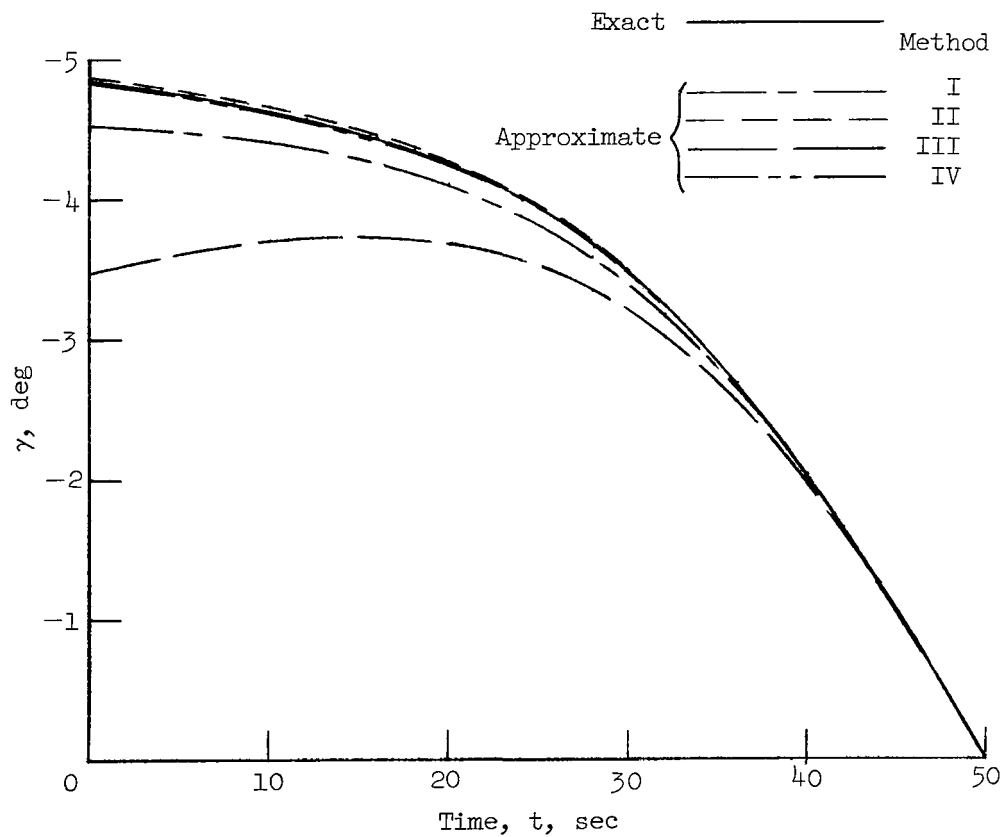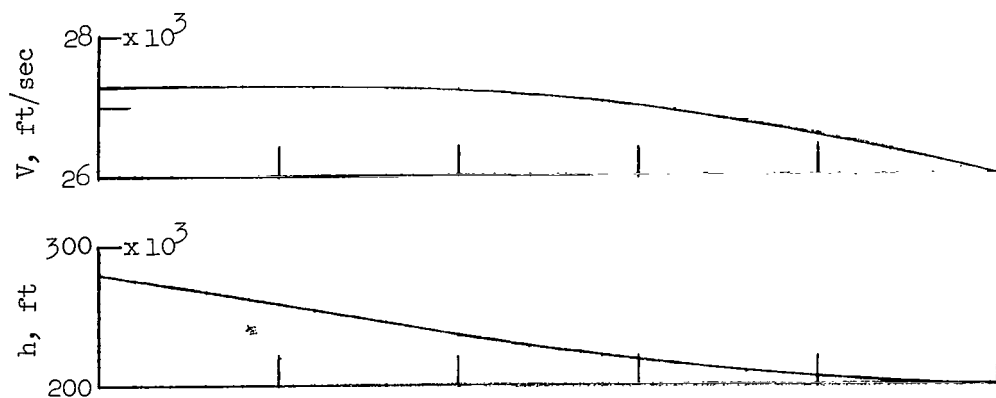
(a) V_d = 26,000 ft/sec.

Figure 5.- Time history of exact trajectory and calculated values of flight-path angle for high
lifting vehicle. h_d = 200,000 feet; C_L = 1.5; γ_d = 0°.
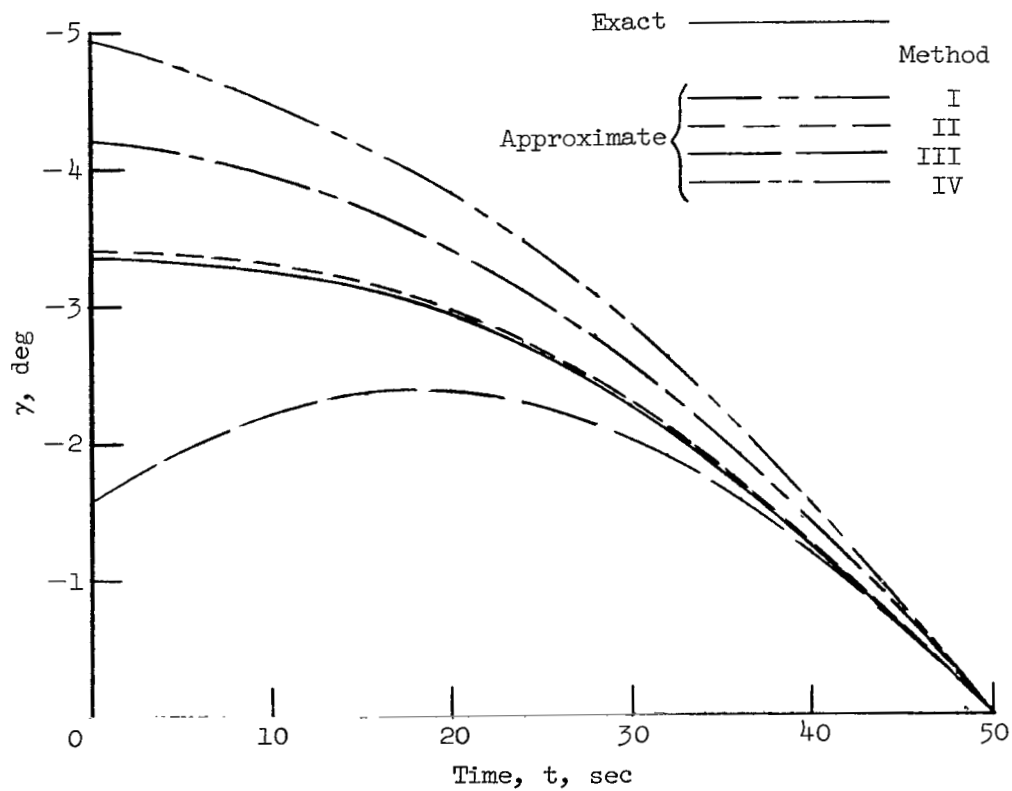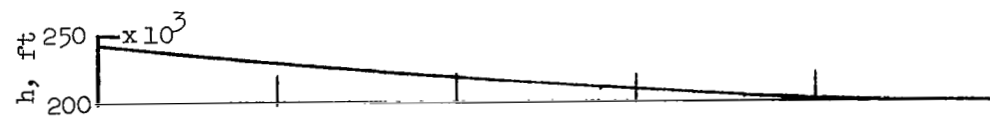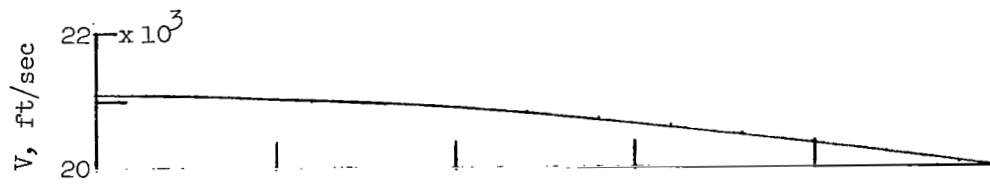
(b) V_d = 20,000 ft/sec.

Figure 5.- Continued.

(c) $V_d = 14{,}000$ ft/sec.

Figure 5.- Concluded.

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